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Ch-4 Real no. as a complete ordered Field

* **Notations (संकेतन) :-**

N = The set of Natural numbers.
(प्राकृत संख्याओं का समुच्चय) Ex. $\{1, 2, 3 \ldots n \ldots\}$

I = The set of Integers
(पूर्णांकों का समुच्चय) Ex. $\{\ldots -2, -1, 0, 1, 2 \ldots\}$

Q = The set of rational numbers.
(परिमेय संख्याओं का समुच्चय) Ex. $\frac{2}{3}$, 15, 3.1414.

R = The set of real numbers.
(वास्तविक संख्याओं का समुच्चय) Ex.

Clearly,

$$N \subset I \subset Q \subset R$$

एक वास्तविक सं. जो परिमेय (rational) नहीं है एक अपरिमेय सं. (Irrational numbers) कहलाती हैं।

उदा :- $\frac{2}{3}$, 15, 3.141414 etc are rational no.

and $\sqrt{2}, \sqrt{5}, 3\sqrt{7}, \sqrt{2}+7, \pi$ etc. are the irrational no.

Ex:-(1) Prove that $\sqrt{2}$ is irrational number.

proof:- Let, us assume the contrary,

$$\sqrt{2} = \frac{p}{q} \quad \{\sqrt{2} \text{ rational no. है}\}$$

Here, $q \neq 0$, and p, q are relatively prime numbers. अर्थात् ये common factor या गुणनखंड नहीं रखते हैं।

Squaring both sides we get,

$$(\sqrt{2})^2 = \left(\frac{p}{q}\right)^2$$

$$\Rightarrow 2 = \frac{p^2}{q^2} \Rightarrow 2q^2 = p^2 \text{ ——— (1)}$$

now q is an integer (पूर्णांक)
So $2q^2$ is also a integer.

Here $p^2 = 2q^2 \Rightarrow \frac{p^2}{2} = q^2$

i.e. p^2, 2 से divide होने वाला एक integer है फलतः p भी 2 से divisible है अन्यथा p^2, 2 से divide नहीं होता।

let $p = 2m$ where m is integer.
by eqn. (1),

$$(2m)^2 = 2q^2 \Rightarrow 4m^2 = 2q^2$$
$$\Rightarrow 2m^2 = q^2 \quad \text{---(2)}$$

समी० (2) से स्पष्ट है कि q भी 2 से divisible है अतः p और q दोनों ही 2 से divisible है, जो इस परिकल्पना (Hypothesis) का विरोध (contradiction) है कि p और q common factor नहीं रखते।

So, $\sqrt{2}$ is irrational no.
Hence proved.

Ex. (2) prove that $\sqrt{8}$ is irrational.

Proof:- माना, $\sqrt{8}$ एक rational no. है।

$$\sqrt{8} = \frac{a}{b}$$

जहाँ a और b दो सह अभाज्य पूर्णांक (Co-prime) हैं

So, $b\sqrt{8} = a$
Squaring both sides,

सह अभाज्य सं.
Co-prime no.
जैसे, $10 = 2 \times 5 \times 1$
$21 = 3 \times 7 \times 1$

HCF = 1

$\Rightarrow b^2 \cdot 8 = a^2$ —— (1) $\Rightarrow b^2 = \frac{a^2}{8}$

$\Rightarrow a^2$ is divisible by 8

So, a is also divisible by 8

let $a = 8c$ {c is integer}

by (1),

$b^2 \cdot 8 = (8c)^2$

$\Rightarrow b^2 \cdot 8 = 64c^2$

$\Rightarrow b^2 = 8c^2 \Rightarrow c^2 = \frac{b^2}{8}$ —— (2)

$\Rightarrow b^2$ is completely divisible by 8

$\because b$ is also —— " ——

by equ. (1) & (2),

अतः a और b दोनों ही 8 से divisible है।

which is the contradiction of our hypothesis

So $\sqrt{8}$ is irrational.

Hence Proved.

Ex. (3) If $a, b, c, d \in R$, $b \neq 0$, $d \neq 0$ and $\frac{a}{b} = \frac{c}{d}$ then prove that $ad = bc$.

Proof:- Given that,

$\frac{a}{b} = \frac{c}{d} \Rightarrow ab^{-1} = cd^{-1}$

$\Rightarrow (ab^{-1}) \cdot d = (cd^{-1}) \cdot d$ {R.H.S. से d का गुणन (multiply) करने पर

$\Rightarrow (b^{-1}a) \cdot d = c(d^{-1}d)$ {by commutative and Associative law

$\Rightarrow (b^{-1}a) \cdot d = c \cdot 1$

$\Rightarrow (b^{-1}a) \cdot d = c$

$\Rightarrow b\,(b^{-1}\cdot a)\cdot d = bc$ { L.H.S. से b का multiply करने पर

$\Rightarrow (bb^{-1})\,(ad) = bc$

$\Rightarrow 1\cdot(ad) = bc$

$\Rightarrow \boxed{ad = bc}$

Hence Proved

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