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* Absolute Value (निरपेक्ष मान):-

मानलो $x \in R$ तो x का निरपेक्ष मान अथवा मापांक (Absolute value or modulus), जिसे हम $|x|$ द्वारा निरूपित करते हैं, नि. प्रकार से define किया जाता है।

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

इस definition का अर्थ यह है कि

$x \geq 0$ होने पर $|x| = x$ तथा
$x < 0$ होने पर $|x| = -x$

Ex:- $|5| = 5$, $|-5| = -(-5) = 5$

Theorem (1): prove that, for each $x \in R$,
$|x| =$ महत्तम $\{x, -x\}$ {greatest = महत्तम}

Proof:- त्रिविकल्पता नियम (law of Trichotomy) से, नि. लि. कथनों में से एक और केवल एक कथन ही सत्य हैं;

(i) $x > 0$ (ii) $x = 0$ (iii) $x < 0$

(i) if $x > 0$ then by the definition of Absolute value, $|x| = x$ —— (1)

$\because 0 < x$ then $-x < 0$

So, $-x < 0$, $0 < x \Rightarrow -x < x$ —— (2)

$\because |x| =$ महत्तम $\{x, -x\}$ by (1) & (2)

(ii) if $x=0$ then, by the definition of Absolute Value, $|x|=0$ —— (3)

$\because x=0$ then $-x=-0=(-1).0=0$

$\text{महत्तम}\{x,-x\} = \text{महत्तम}\{0,-0\}$

$= \text{महत्तम}\{0,0\}$

$=0=x$ —— (4)

by equ. (3) & (4),

$$|x| = \text{महत्तम}\{x,-x\}$$

(iii) if $x<0$ then, by the defi.^{n} of Absolute Value.

$|x| = -x$ —— (5)

$\because x<0$ then $0<-x$,

$x<0,\ 0<-x \Rightarrow x<-x$ —— (6)

So, $\text{महत्तम}\{x,-x\} = -x$

by equ. (5), (6)

$$|x| = \text{महत्तम}\{x,-x\}$$

Theorem (2): for each $x \in R$, prove that,

(a) $|x| \geq 0$ (b) $x \leq |x|$ and $-x \leq |x|$

(c) $|x| = |-x|$

Proof:- (a) Case I. when $x \geq 0$

by the defi.^{n} $|x|=x$ then

$\because |x|=x$ and $x\geq 0 \Rightarrow |x| \geq 0$

Case II when $x<0$

$|x|=-x$, $\because x<0$ then $-x>0$,

now

$|x|=-x$ and $-x>0 \Rightarrow |x|>0$

$\Rightarrow |x| \geq 0.$

by Case I, and II,

$$|x| \geq 0.$$

(b) we know that by theorem (1)

$|x| =$ महत्तम $\{x, -x\}$ —— (1)

किंतु महत्तम $\{x, -x\} \geq x$

तथा महत्तम $\{x, -x\} \geq -x$ —— (2)

by equ. (1) & (2)

$|x| \geq x$ and $|x| \geq -x$ i.e.

$\Rightarrow x \leq |x|$ and $-x \leq |x|$

(c) By theorem (1), $|x| =$ महत्तम $\{x, -x\}$

$=$ महत्तम $\{-x, -(-x)\}$ $\{\because -(-x) = x\}$

$|x| = |-x|$

Hence Proved.

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Theorem (3): If $x, y, z \in R$ then prove that,

(a) $|x-y| = |y-x|$

(b) $|x+y| \leq |x| + |y|$

(c) $|x-z| \leq |x-y| + |y-z|$

(d) $||x| - |y|| \leq |x-y|$

proof:- (a) we know that, by theorem (2) c, & put $x-y$ in the place of x,

$|x-y| = |-(x-y)| = |(-x)+y|$

$\boxed{|x-y| = |y-x|}$

(b) By theorem (2) b,

$x \leq |x|$ and $y \leq |y|$

$\therefore$

$x+y \leq |x| + |y|$ —— (1)

again by theorem 2(b),

$-x \leq |x|$ and $-y \leq |y|$

$\therefore (-x) + (-y) \leq |x| + |y|$ —— (2)

by equ. (1) & (2),

महत्तम $\{x+y, -(x+y)\} \leq |x| + |y|$ —— (3)

by theorem (1), $|x+y| =$ महत्तम $\{x+y, -(x+y)\}$

$$\boxed{|x+y| \leq |x| + |y|}$$

(c) By (b), $|x+y| \leq |x| + |y|$

put $x = x-y$

$y = y-z$

then,

$|(x-y)+(y-z)| \leq |x-y| + |y-z|$

$$\Rightarrow \boxed{|x-z| \leq |x-y| + |y-z|}$$

(d) we know that, $|x| = |(x-y)+y|$

$|x| \leq |x-y| + |y|$

$|x| - |y| \leq |x-y|$ —— (4)

interchange x and y in equ. (4),

$|y| - |x| \leq |y-x| = |x-y|$

but, $|y| - |x| = (|x| - |y|)$

so,

$-(|x| - |y|) \leq |x-y|$ —— (5)

by equ. (4) & (5)

महत्तम $\{|x| - |y|, -(|x| - |y|)\} \leq |x-y|$ —— (6)

by equ. (6) and Theorem (1) we've,

$$\boxed{||x| - |y|| \leq |x-y|}$$

Hence Proved