distinct → different

B.Sc. VI Sem "Maths" 13/4/20

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IMP **Complete Metric Space :-** A metric space (X, d) is called complete metric space if, each cauchy sequence in it convergence to a point of X.

Result :-

if convergent sequence has infinitely many distinct terms then limit of the sequence is also the limit point of range set of the sequence.

proof :- Let,

convergent sequence $\{x_n\}_{n=1}^{\infty}$ has infinitely many distinct points.

Let,

$$\lim_{n \to \infty} x_n = p$$

we have to prove,

p is also the limit point of the range set,

$$A = \{x_n : n \in N\}$$

or $= \{x_1, x_2, x_3 \text{-------}\}$

Assume Contrary (उल्टा),

Let p be not the limit point of range set A.

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then $\exists$ an open sphere $S_\epsilon(p)$ {the nbd of p}
Such that,

$$S_\epsilon(P) \cap A = \phi \text{ or } S_\epsilon(P) \cap A = \{p\}$$

Since, $\lim_{n\to\infty} x_n = p$

therefore above $\epsilon > 0$, $\exists$ a +ve integer m such that,

$$x_n \in S_\epsilon(p) \text{ for all } n \geq m$$

(by the defi. of limit of sequence).

This is a contraduction. therefore p is the limit point of the range set A.

{ i.e. $S_\epsilon(p)$ Contains infinite distinct elements of set A. }

Therefore p is the limit point of range set A.

Hance proved.

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Subgroup - Subgroup is group which holds all the property of group.

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Result:- Every closed subspace of a complete metric space is complete.

proof:-

Let,

(Y, d) be a closed sub space of complete metric space (X, d).

we have to prove sub space (Y, d) complete.

Let,

$\{y_n\}_{n=1}^{\infty}$ be a cauchy sequence of Y.

Since $Y \subseteq X$

Therefore $\{y_n\}$ will be a cauchy sequence of X.

Since,

(X, d) is complete therefore cauchy sequence $\{y_n\}$ will converge to a point p (say). of X.

$$\lim_{n \to \infty} y_n = p \in X$$

following two cases are possible,

Case I :- if range set A of seq. $\{y_n\}$ contains finite no. of distinct points.

i.e. $A = \{y_1, y_2, y_3 \cdots y_n, p, p, p \cdots\}$

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$A \subset Y$
$\Rightarrow p \in Y$

Case II :- if range set A contains infinitely many distinct points.

we know that, If convergent Sequ. has infinitely many distinct points then limit of the sequence is also the limit point of range set of the sequence.
Therefore,
p is the limit point of range set A.
since, $A \subset Y$
Therefore p will also be the limit point of Y.
Since Y is closed therefore $p \in Y$
(a closed set contains all its limit point).

This shows that Cauchy Sequ. $\{y_n\}$ of Y are convergence to a point p of Y therefore subspace (Y, d) is complete.

Hance proved.