B.Sc. IV Sem "Maths"
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* **Nature of bilinear Transformation :-**

(1) Every bilinear transformation with two fixed points α, β can be put in the form -

$$\boxed{\frac{w-\alpha}{w-\beta} = \lambda \cdot \frac{z-\alpha}{z-\beta}}$$

(i) If $|\lambda| = 1$ elliptic (दीर्घवृत्तीय)

(ii) If $\lambda > 0$ $(\neq 1)$ hyperbolic (अतिपरवलयिक)

(iii) If λ is neither real nor $|\lambda| = 1$ then it is case of Loxodromic
कुछ दिशा, (नौ पथीय)

(2) Every bilinear transformation which has only one finite fixed point α. Can be put in the form.

$$\boxed{\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + d}$$

In this case transformation is a parabolic (परवलयिक)

* Find the fixed point and normal form (स्थिर बिंदु और प्रसामान्य रूप) of the following bilinear transformation and discuss the nature of these transformation (elliptic, hyperbolic, parabola.).

(i) $w = \dfrac{z}{2-z}$

Solⁿ :- Here $w = \dfrac{z}{2-z}$

The fixed point are given by:

$$z = \frac{z}{2-z} \quad \{w \text{ के स्थान पर } z \text{ रखने पर}\}$$

$\Rightarrow \quad 2z - z^2 = z$

$\Rightarrow \quad z^2 - z = 0$

$\Rightarrow \quad z(z-1) = 0$

$\Rightarrow \quad z = 0, 1$ (fixed Points)

Let $\alpha = 0$, $\beta = 1$

$\therefore$ Here $w = \dfrac{z}{2-z}$

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now for the normal form as, $\frac{w-\alpha}{w-\beta}$

Here $\alpha = 0$, $\beta = 1$ & $w = \frac{z}{2-z}$

now,

first we find $w-\alpha$ i.e.

$$w - 0 \Rightarrow \frac{z}{2-z} - 0 \Rightarrow w = \frac{z}{2-z}$$

and then we find $w-\beta$ i.e.

$$w - 1 \Rightarrow \frac{z}{2-z} - 1 \Rightarrow \frac{z - 2 + z}{2-z}$$

$$\Rightarrow \frac{2z-2}{2-z}$$

now, $\frac{w-\alpha}{w-\beta}$ is, $\frac{w-0}{w-1} = \frac{\frac{z}{\cancel{2-z}}}{\frac{2z-2}{\cancel{2-z}}}$

$$= \frac{z}{2z-2}$$

$$= \frac{z}{2(z-1)}$$

It is in the form,

$$\frac{w-\alpha}{w-\beta} = \lambda \cdot \frac{z-\alpha}{z-\beta}$$

where $\lambda = \frac{1}{2} > 0 \ (\neq 1)$

$\Rightarrow$ So it is Hyperbolic. Ans.

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(2) $$w = \frac{3z-4}{z-1}$$

Sol^n:- For the fixed point put $z = w$

$$z = \frac{3z-4}{z-1} \Rightarrow z^2 - z = 3z - 4$$

$$\Rightarrow z^2 - 4z + 4 = 0$$

$$\Rightarrow (z-2)^2 = 0$$

$$\Rightarrow \boxed{z = 2, 2} \text{ (fixed points)}$$

Here $\alpha = 2$, $\beta = 2$ (both are same)

So Here is the only one fixed point.

$\Rightarrow$ The Transformation is parabolic.

Ans.

(3) $$w = \frac{z-1}{z+1}$$

Sol^n:- For the fixed Point Put $z = w$.

$$z = \frac{z-1}{z+1} \Rightarrow z^2 + z = z - 1$$

$$\Rightarrow z^2 + 1 = 0 \Rightarrow \boxed{z = \pm i}$$

Hence

$+i$ and $-i$ are two fixed points in this case.

Normal Form :- $\frac{w-\alpha}{w-\beta}$ Here $\alpha = i$, $\beta = -i$

now $w-\alpha = \frac{z-1}{z+1} - i$

& $w-\beta = \frac{z-1}{z+1} + i$

So, $\frac{w-\alpha}{w-\beta} = \frac{w-i}{w+i} = \frac{\frac{z-1}{z+1} - i}{\frac{z-1}{z+1} + i}$

$= \frac{z-1-iz-i \,/\, \cancel{z+1}}{z-1+iz+i \,/\, \cancel{z+1}}$

$= \frac{(1-i)(z-i)}{(1+i)(z+i)}$

$= \frac{(1-i)^2 (z-i)}{(1+i)(1-i)(z+i)}$ $\left\{ (1-i)\text{ से अंश और हर में गुणा} \right.$

$= -i \frac{(z-i)}{(z+i)}$

which is required normal form like

$\frac{w-\alpha}{w-\beta} = \lambda \frac{z-\alpha}{z-\beta}$

where $\lambda = -i \Rightarrow |\lambda| = \sqrt{(-1)^2} = 1$

$\Rightarrow$ Elliptic Ans